Uncertainty Analysis in the Evaluation of PMSM End-Winding Inductance

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Abstract—This paper presents a comparison between several numerical and analytical methods used to evaluate the End-Winding Inductance (EWI) of three phase stator windings of induction or synchronous machines. A prototype of a Permanent Magnet Synchronous Machine (PMSM) is considered as a case study for the illustration of the computational and experimental approaches. The numerical simulations are carried out using two Finite Element Method (FEM) based professional software packages Flux and Comsol. This study proves that the errors generated by the classical analytical approach could be important due to the poor approximation of magnetic field lines distribution at the stator end-winding zone. It is also proved that the accurate evaluation of EWI in the analysis of PMSM becomes of paramount importance in case of short-length large-diameter PMSM. Experimental measurement results obtained on a PMSM prototype are compared against numerical ones.

I. INTRODUCTION

One of the most efficient electrical machines that is increasingly used in industry, both as motor and generator, is the Permanent Magnet Synchronous Machine (PMSM). Compared to the electromagnetic excitation synchronous machine, PMSM is characterized by higher efficiency and reliability, higher energy density, overall reduced size, etc.

An important phase during the design and analysis of PMSM consists in the evaluation of the electrical parameters of the stator windings.

The PMSM stator windings consist typically of an active part, i.e. the winding part situated in the stator slots and an inactive part that corresponds to the frontal end-winding region. The frontal part has no contribution to the electromechanical conversion process that takes place in the machine.

The PMSM stator phase windings are characterized by an electrical resistance that can be easily evaluated experimentally or by analytical calculations and a sum of inductances that are more difficult to be evaluated or measured individually.

The stator winding total inductance (synchronous inductance) \( L \) per phase is composed of a linkage armature inductance \( L_L \) which corresponds to the armature reaction magnetic field that reaches the rotor armature and a leakage inductance \( L_\sigma \) associated to the armature reaction magnetic flux (leakage flux) which does not reach the rotor armature:

\[
L = L_L + L_\sigma
\]  

(1)

The leakage inductance \( L_\sigma \) at its turn can be divided into slot leakage inductance, tooth-tip leakage inductance, differential leakage inductance and end-winding leakage inductances.

All the stator winding inductances (both linkage and leakage) can be determined with relatively good accuracy analytically or numerically by 2D Finite Element (FE) analysis of PMSM except the last one, i.e. End-Winding Inductance (EWI).

The analytical evaluation of EWI can be affected by large errors due to the complicated magnetic flux distribution at the end winding region of the PMSM approximated by simplified analytical models.

A 2D FE based evaluation of EWI is not possible because the end-winding region is not included in the machine cross-section which is typically the computation domain selected for a 2D magnetic field analysis. Therefore the only suitable way to evaluate accurately the EWI is by using a full 3D FE model of PMSM.

While an accurate evaluation of EWI is more or less important in case of axially long PMSMs, it becomes of paramount importance for short-length large-diameter PMSM where this parameter may exceed half of the total leakage inductance of the machine [1]-[2]. Such examples of PMSMs are used as low speed high torque machines in robotics, machine tools, direct drive wind turbines etc. In case of PMSM operating as generator, EWI should be accurately evaluated as well since it affects directly the voltage regulation index (difference between the magnitudes of no-load and full-load voltages divided by the rated voltage) of the machine.

This paper presents a comparison between several analytical and numerical methods used to evaluate the EWI in case a short-length large-diameter PMSM, with emphasis on the computation uncertainty and relative differences. Numerical results are compared with experimental measurements.

II. DESCRIPTION OF STUDIED PMSM PROTOTYPE

The physical support for this study is a small power, three-phase PMSM prototype with double layer winding, Fig. 1, characterized by the main data synthetically presented in Table I. The machine shown in Fig. 1 is mounted in an aluminum frame, suitable for experimental tests.
TABLE I

<table>
<thead>
<tr>
<th>MAIN DATA OF STUDIED PMSM OPERATING AS GENERATOR</th>
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<tbody>
<tr>
<td>Rated power [W]</td>
</tr>
<tr>
<td>Rated speed [rpm]</td>
</tr>
<tr>
<td>Rated voltage [V]</td>
</tr>
<tr>
<td>No. of stator slots</td>
</tr>
<tr>
<td>No. of permanent magnets</td>
</tr>
<tr>
<td>Air-gap length [mm]</td>
</tr>
<tr>
<td>Stator inner diameter [mm]</td>
</tr>
<tr>
<td>Stator outer diameter [mm]</td>
</tr>
<tr>
<td>Slot height [mm]</td>
</tr>
<tr>
<td>Laminations stack length [mm]</td>
</tr>
<tr>
<td>No. of turns per phase</td>
</tr>
<tr>
<td>No. of coils per phase (pole pitch length)</td>
</tr>
<tr>
<td>Axial distance from coil end to laminations stack [mm]</td>
</tr>
</tbody>
</table>

III. IMPORTANCE OF EWI EVALUATION FOR 2D FEM ANALYSIS OF PMSM

PMSM can be comprehensively analyzed using 2D or 3D Finite Element (FE) based field-circuit coupled models. The 3D numerical models generally offer more consistent information in comparison with 2D models but they are far more costly in terms of problem description complexity and computation effort. That is why in many cases the study of PMSM is carried out using 2D models rather than 3D ones or an economical combination of the two.

A comprehensive analysis of the working characteristics of PMSM can be envisaged only if the FE model is coupled with the circuit model of the machine. The main reason for using a field-circuit model is that the induced currents in the PMSM stator windings that are not a-priori known. They can be indirectly computed only by associating a circuit model to the FE model of the machine.

An economical approach considers the FE reduced 2D model, Fig. 2a, coupled with the three-phase circuit model of the PMSM, Fig. 2b. It is possible to take into account the front ends of the stator winding in the 2D model, only by adding suitable circuit elements to the field model.

The elaboration of the 2D transient electromagnetic field-circuit coupled model of the PMSM starts by drawing the cross-section geometry of the machine and building the FE mesh, Fig. 2a; physical properties are subsequently assigned. The electric circuit of stator winding is generated and correlated with the FE model. The circuit parameters in Fig. 2b are: RU, RV, RW - phase resistances, BU1, BU2 ... BW2 - positively and negatively oriented coil sides of stator windings, LSU, LSV, LSW - front end winding inductances, S1 ... S3 - switches, RLU, RLV, RLW - load resistances, RUV, RVW, RUW - voltmeters.

The EWI (LSU, LSV, LSW) are the only parameters in the diagram susceptible to be affected by large uncertainties, especially when simplified analytical formulas are used.

The 2D FE model of the PMSM is governed by the following partial differential equation expressed in magnetic vector potential formulation \(A\):

\[
\text{curl} \left[ \frac{1}{\mu} \text{curl} A \right] - Hc = J_s
\]

where \(\mu\) is the magnetic permeability of the materials, \(Hc\) is the coercitive magnetic field strength and \(J_s\) is the source current density that is a-priori unknown but it can be computed by the field - circuit coupled model Fig. 2b.

Fig. 2. 2D FE model of studied PMSM; a) FE computation domain and mesh; b) Circuit model of PMSM associated to the FE model.
IV. COMPUTATION METHODS USED TO EVALUATE EWI

A. Analytical approach

The Analytical Approach (AA) used for the evaluation of EWI is based on a formula that depends on the winding type. For a double layer winding, as in our case, the calculation of per phase EWI can be carried out using the following formula [3]:

\[ L_{ew} = 0.34 \frac{2\mu_0 w_1^2}{p} \left( L_{ew} - 0.64 \tau \right) \]  

where \( \mu_0 \) is the magnetic permeability of vacuum, \( w_1 \) is the number of turns per phase, \( p \) is the number of pole pairs, \( L_{ew} \) is the length of average wire at the end-winding zone, \( \tau \) is the pole pitch length.

B. Numerical approaches. 3D FE analysis

A more accurate evaluation of EWI can be done by using a 3D FEM analysis of PMSM, since it takes into account the complicate shape of magnetic field at the stator end-winding zone.

An approximate geometry of the stator armature that includes the laminations stack and the stator windings is presented in Fig. 3. The rotor is removed during the 3D FEM simulations since its influence on the EWI is considered negligible.

Since the stator magnetic core is far from saturation under given supply conditions, the 3D numerical computations are carried out for constant magnetic permeability \( \mu_r = 1000 \) of the stator core. Thereby, the field problem in this case is linear of magnetostatic type.

The exploitation of the axial physical symmetry of the device and the superposition principle allowed us to take into account only half a stator coil, Fig. 4.

The 3D magnetostatic field problem solved with the Flux software package is governed by partial differential equations expressed in magnetic vector potential \( \mathbf{A} \).

In Flux3D the magnetic vector potential formulation \( \mathbf{A} \) is used in all the regions (magnetic or not) of the computation domain, the corresponding partial differential equation having the same form as (2).

The electromagnetic field analysis in Comsol is based on the \( \mathbf{A}V \) formulation (magnetic vector potential \( \mathbf{A} \) and electric scalar potential \( V \)) in the coil region and on the \( \mathbf{A} \) formulation elsewhere [4]. The \( \mathbf{A}V \) formulation is governed by the following partial differential equations:

\[ -\text{div} \left[ \sigma \cdot \text{grad} \ V - \mathbf{J}_s \right] = 0, \]
\[ \text{curl} \left[ (1/\mu \ \text{curl} \mathbf{A}) - \mathbf{H}_f \right] + \sigma \cdot \text{grad} \ V = \mathbf{J}_s, \]

The computation domain is bounded by a horizontal plane of symmetry characterized by non-zero tangential magnetic flux density and normal current density (corresponding to the boundary conditions \( \mathbf{B} \cdot \mathbf{n} = 0; \ \mathbf{J} \times \mathbf{n} = 0 \)). That symmetry plane represents a cross section through the middle of the studied machine.

The open boundary is modeled in Flux3D by means of a special entity called Infinite Box. This entity is based on a Kelvin transformation that converts the exterior infinite space into a limited one [5].
The 3D computation domain in Comsol is magnetically insulated \((\mathbf{B} \cdot \mathbf{n} = 0)\) on all the boundary surfaces.

The EWI computations are carried out using two approaches. The first approach named Numerical Approach 1 (NA1) considers only one phase of the stator winding fed with a constant current \(I_{m}\). After solving the 3D magnetostatic field problem, the EWI \((L_{ew})\) is computed starting from the total magnetic energy \(W_{airext}\) stored in the exterior of the PMSM:

\[
W_{airext} = \int_{V_{H}} \left( \int_{0}^{H} dB \right) dV
\]  

(5)

Then the EWI is computed with the formulas below where \(K\) represents a multiplication coefficient that takes into account the symmetries and the superposition effect:

\[
W_{airext} = L_{ew} I_{m}^2 / 2
\]  

(6)

\[
L_{ew} = 2 K W_{airext} / I_{m}^2
\]  

(7)

The Numerical Approach 2 (NA2) consists in computing the inductance per phase \(L_t\) by solving subsequent 3D FEM problems for decreasing lengths of the stator laminations stack [6]. The inductance is then computed from the total energy \(W\) stored in the FEM computation domain and uses the relations below where \(K\) is the same coefficient described above:

\[
W = \int_{V_{H}} \left( \int_{0}^{H} dB \right) dV
\]  

(8)

\[
W = L_t I_{m}^2 / 2
\]  

(9)

\[
L_t = 2 K W / I_{m}^2
\]  

(10)

When the stator laminations stack length tends to zero, the total inductance \(L_t\) will become \(L_{ew}\).

V. NUMERICAL RESULTS AND DISCUSSIONS

The results of EWI computed using the analytical and numerical methods described above are presented in Table II. Numerical computations are carried out using both Flux and Comsol software packages. We can notice large differences between the numerical and analytical results. The relative difference obtained in case of numerical approach NA1 reaches 38.84 % (Flux computation) and 43.13 % (Comsol computation) respectively, the numerical results being larger than the analytical ones. In contrast, the results obtained by the proposed numerical methods are much closer to each other.

Fig. 5 presents the dependence between the total magnetic energy \(W\) stored in the computation domain and the length of the magnetic core \(L_{core}\). The energy \(W\) is composed of the magnetic energy stored in the exterior of the machine \(W_{airext}\) (practically a constant quantity, independent on the stator core length) and the energy in the magnetic core region \(W_{core}\) that is a linear function of the stator core length \((W_{core} = C_t L_{core})\). By extending the line \(W = f(L_{core})\) to the intersection point with the abscissa axis, we can estimate the magnetic energy at zero stator core length. That point represents in fact the magnetic energy \(W_{ew}\), that corresponds to the end-winding zone of the machine and used for the calculation of EWI by the NA2 [4].

Fig. 5 illustrates also at other scale \((L_t \sim W)\) the weight of the EWI from the total inductance of a phase winding. For the short magnetic core of the PMSM studied here \((L_{core} = 15 \text{ mm})\), EWI represents 38.68% of the total inductance, which confirms how important it is to have an accurate estimation of its value, when the circuit analysis of the machine is performed.

<table>
<thead>
<tr>
<th>Computation method</th>
<th>(L_{ew} [\text{mH}])</th>
<th>Relative difference 100(NAx-AA)/NAx [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA</td>
<td>0.1308</td>
<td>-</td>
</tr>
<tr>
<td>NA1</td>
<td>0.2130</td>
<td>38.60</td>
</tr>
<tr>
<td></td>
<td>0.2230</td>
<td>41.35</td>
</tr>
<tr>
<td>NA2</td>
<td>0.2000</td>
<td>34.60</td>
</tr>
<tr>
<td></td>
<td>0.2300</td>
<td>43.13</td>
</tr>
</tbody>
</table>

VI. EXPERIMENTAL RESULTS

An experimental test was conducted on the PMSM prototype and the measured results were compared to numerical values. During this test the rotor was removed from the stator armature, under the assumption that EWI is not influenced by the presence of the rotor.

Following the standardized experimental procedure described in [7], we determined the armature leakage reactance \(X_a\) by the so called applied voltage test with the rotor removed. The leakage reactance \(X_a\) includes the EWI but also other types of leakage reactances that could not be separated during the experiment.

For this experiment a search coil with an opening equal with the pole pitch is mounted on the inner surface of the stator, Fig. 6. The induced voltage \(U_C\) measured across the search coil is directly related to the magnetic flux across the armature active surface in the space that is normally occupied by the rotor.

The PMSM three-phase stator winding has been supplied with a symmetrical voltage system and the standard electric circuit quantities (active power - \(P\), phase current - \(I\) and phase voltage - \(U\)) are determined. The impedance \((Z)\), the electric resistance \((R)\) and the total reactance \((X)\) of a phase winding are then determined by the customary relations:

\[
Z = \frac{U}{I}, \quad R = \frac{P}{3I^2}, \quad X = \sqrt{Z^2 - R^2}
\]  

(11)
where $X$ is the total reactance of the stator phase winding under conditions specified above and it includes self and mutual reactances of one phase winding relative to the other phases.

The reactance $X$ cumulates the contributions of the magnetic flux associated to the leakage phenomena in normal working conditions ($X_{\sigma}$) and of the flux across the armature active surface, created by the armature winding in the space which is normally occupied by the rotor ($X_b$):

$$X = X_{\sigma} + X_b$$  \hspace{1cm} (12)

The reactance $X_b$ is computed using the formula [7]:

$$X_b = \frac{U_C \cdot N k_w}{I \cdot N_C},$$  \hspace{1cm} (13)

where $N$ is the number of turns per phase (in our case $N = 180$), $k_w$ is the armature winding factor (in our case $k_w = 1$), $N_C$ is the number of turns of the search coil (in our case $N_C = 11$).

A comparison between numerical results and experimental ones was carried out by developing two FE models in Flux and Comsol software packages respectively.

The measured results in case of the three-phase AC voltage supplied PMSM are presented in Table III. The experimental results of the reactance $X_b$ are compared with the numerical determinations.

The relative differences between numerical and experimental results are of about 7.43 \% in case of Flux simulations and about 3.18 \% in case of Comsol simulations. The differences between the results could come from the measurement errors at low voltages, from some small unbalance of the phase currents in the experiment, from the limitations in modeling the real electromagnetic structure and physical properties, etc.

**TABLE III**

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</thead>
<tbody>
<tr>
<td>3.22</td>
<td>3.73</td>
<td>15.7</td>
<td>0.375</td>
<td>0.497</td>
<td>0.326</td>
<td>0.0377</td>
</tr>
</tbody>
</table>

**TABLE IV**

<table>
<thead>
<tr>
<th>$X_b$ [Ω]</th>
<th>Relative difference 100(Num. - Exp.)/Exp. [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experim.</td>
<td>Comsol</td>
</tr>
<tr>
<td>0.0377</td>
<td>0.0389</td>
</tr>
</tbody>
</table>

Fig. 6. Experimental model PMSM used for measurements (stator with rotor removed).

Fig. 7. Magnetic field lines and flux density on the machine cross section; a) Flux results; b) Comsol results.

VII. CONCLUSION

This study is focused on the uncertainty analysis of EWI computation of a PMSM. An analytical and several numerical approaches were used to compute EWI and the results were compared. Since the differences between numerical and analytical results were significant, we can draw the conclusion that the traditional analytical formulas of EWI should be used with precaution.
Based on the obtained results we can state that in case of short-length large-diameter PMSMs, when a good accuracy of numerical results is required, a 3D FEM based numerical evaluation of EWI is recommended.

An experimental analysis of a PMSM prototype was carried out and the measured results were compared with numerical ones. The differences between the two sets of results are relatively small.

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